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Bubble diameter correlation via numerical experiment

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Abstract

A numerical experiment has been carried out to find a theoretically sound correlation relating mean bubble diameter in the main section of a bubble column to the liquid physicochemical properties (density, viscosity, surface tension), and the superficial gas velocity. A correlation has been proposed in the form:

$$d_{32} = 0.289\rho_{\mathrm{L}}^{-0.552}\mu_{\mathrm{L}}^{-0.048}\sigma^{0.442}U_{\mathrm{G}}^{-0.124}$$

or, in an approximate, dimensionless form:

$$Fr = 0.6\left(\frac{U_{\mathrm{G}}^4\rho_{\mathrm{L}}}{g\sigma}\right)^{1/2}$$

The values obtained from the above correlations are in good agreement with experimental values, obtained for seven organic liquids.

Keywords: Bubble column; Bubble diameter; Mathematical modeling

1. Introduction

Bubble diameter is an important parameter in the design of mass exchangers and chemical reactors using gas bubbling as a means of gas–liquid contacting. A number of empirical and semi-empirical correlations, enabling the calculation of this parameter, exists in the literature. However, their predictions regarding the influence of liquid phase properties (density, viscosity, surface tension) are highly divergent. This is caused by the fact that it is impossible to change any of the above properties without changing the other two, and moreover, the range of changes realizable using easily accessible liquids is very limited. Yet in many cases a prediction of bubble diameter is necessary for a liquid, whose properties are outside the ranges investigated. In other cases, extrapolation is necessary to the conditions well outside the validity range of the existing correlations. To overcome this difficulty, and to develop a theoretically based correlation for bubble diameter, a numerical experiment has been used in this work.

2. The bubble column model

In any bubble column four different regions may be distinguished [1]:

- the region of primary bubbles;
- the region of secondary bubbles;
- the region of dynamic equilibrium between coalescence and disruption of bubbles;
- the separation region.

In sufficiently deep liquid layers, e.g. in bubble columns, the third (equilibrium between coalescence and disruption) region occupies most of the column volume. This region is the subject of the present work.

The distribution of the bubble diameters can be described by the number distribution density function $f_{\mathrm{N}}(d)$. According to the most of the experimental data, for pure liquids this function can be described by the log-normal distribution:

$$f_{\mathrm{N}}(d) = \frac{1}{d\sqrt{2\pi}}\exp\left\{-\frac{1}{2\sigma_{\mathrm{nl}}^2}[\ln(d) - \mu_{\mathrm{nl}}]^2\right\} \quad (1)$$

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Nomenclature

A	constant in Eq. (10)
B	break-up rate ($m^{-3}\,s^{-1}$)
C	coalescence rate ($m^{-3}\,s^{-1}$)
d	bubble diameter (m)
d_{32}	Sauter bubble diameter (m)
D	column diameter (m)
f_N	distribution density function (m^{-1})
Fr	Froude number ($Fr = U_G^2 d_{32}^{-1} g^{-1}$)
g	gravity acceleration ($m\,s^{-2}$)
G	generation function ($m^{-3}\,s^{-1}$)
K	constant in Eq. (12)
n	bubble concentration per unit volume (m^{-3})
t	time (s)
u	bubble rise velocity (s^{-1})
U_G	superficial gas velocity (with respect to the total cross-section of column) ($m\,s^{-1}$)
z	axial co-ordinate

Greek letters

α	exponent in Eq. (10)
β	exponent in Eq. (10)
γ	exponent in Eq. (10)
δ	exponent in Eq. (10)
ε	gas hold-up
μ	liquid viscosity (Pa s)
μ_{nl}	parameter of log-normal distribution in Eq. (1)
ρ	liquid density ($kg\,m^{-3}$)
σ	surface tension ($N\,m^{-1}$)
σ_{nl}	parameter of log-normal distribution in Eq. (1)
υ	bubble volume (m^3)

Subscripts

i, j, k, l, m	bubbles belonging to class i, j, k, l, m
L	liquid

In this work the Sauter mean bubble diameter was used. It is defined by Eq. (2):

$$d_{32} = \frac{\sum_{i=1}^{N} d_i^3}{\sum_{i=1}^{N} d_i^2} \tag{2}$$

The diameter d_{32} was calculated using the function $f_N(d)$ according to Eq. (3):

$$d_{32} = \frac{\int_0^\infty d^3 f_N(d)\delta d}{\int_0^\infty d^2 f_N(d)\delta d} \tag{3}$$

In our earlier work [2], the bubble distribution function was determined for a number of liquids using a theoretical model, based on the bubble population balance equation in the form suggested by Fleisher et al. [3]:

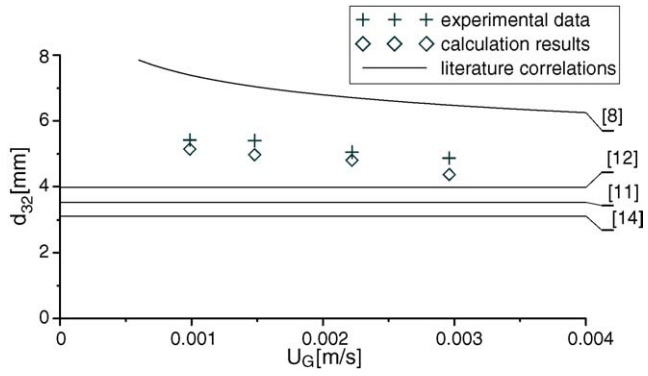

Fig. 1. Dependence of the Sauter bubble diameter on the saturated gas superficial velocity for the air–cyclohexane system.

$$\frac{\partial}{\partial t} n(z,d,t) + \frac{\partial}{\partial z}\left[n(z,d,t)u(z,d)\right] + \frac{\partial}{\partial d}\left[n(z,d,t)\frac{\partial}{\partial t} d(z,d)\right] = G(z,d,t) \tag{4}$$

where the first term describes the change of bubble number concentration with time, the second is the convection term, the third describes bubble growth, and the right hand side is the generation function. For the equilibrium region considered in this work, we observed experimentally that the bubble size distribution does not change in time or along the column axis [4]. Moreover, in the absence of mass transfer and with sufficiently small pressure change, one can assume that all the terms on the left hand side are equal to zero. Dividing the total bubble population into N classes one can write Eq. (4) as:

$$G_i = 0 \tag{5}$$

where G_i is the generation function for bubbles of class "i".

The generation function is the difference between bubble birth and death functions. The bubble "births" in a given class result from breaking a bigger bubble, or from the coalescence of smaller bubbles. Assuming that a bubble can be broken into two smaller bubbles of equal volume (which is rather arbitrary assumption) or be formed by coalescence of two smaller ones, we can write:

$$G_i = \frac{1}{2}\sum_{k=1}^{N}\sum_{l=1}^{N} C_{i,kl} - \sum_{j=1}^{N} C_{ij} + 2B_m - B_i \tag{6}$$

where

$$\upsilon_m = 2\upsilon_i \tag{7}$$

and

$$C_{i,kl} = \begin{cases} C_{kl} \text{ if } \upsilon_k + \upsilon_l = \upsilon_i \\ 0 \text{ if } \upsilon_k + \upsilon_l \neq \upsilon_i \end{cases} \tag{8}$$

Of course one can consider other, more complicated scenarios than simple bi-molecular events. However, their validity is still open to discussion [5], and the simple model assumed here gives very good agreement with the experiment [2] (Fig. 1).

The model of the coalescence/redispersion processes in bubble columns used in this work was that originally suggested by Prince and Blanch [6].

The model assumes that:

- the bubble coalescence rate is equal to the product of the bubble collision rate and the collision efficiency;
- the bubble collisions may be caused by turbulence, buoyancy or laminar shear;
- the bubble break-up rate is equal to the product of collision rate of bubbles and turbulent eddies and collision efficiency;
- bubbles are broken by eddies of the same size as the bubble or smaller (but not smaller than 20% of bubble diameter);
- the bubble-eddy collision efficiency depends on the eddy kinetic energy.

The details of the model used and the results obtained using this model have been described in our two earlier papers [2,4]. They show very good agreement between the results of calculations and those obtained experimentally, even when extrapolated to higher temperatures and pressures [2]. It may be noted here that there exists a number of models, describing gas–liquid flows at different scales (interface tracking models for single bubble, Euler–Lagrange models for bubble swarms, Euler–Euler models for the whole apparatus). These models can be coupled to give a multi-level model [7]. However, many of them require introduction of the bubble diameter in the input data, which makes them useless for our purpose.

3. The numerical experiment

In order to carry out the numerical experiment, we imagined three sets of "virtual liquids", each set having two physicochemical parameters constant and different values of the third parameter. The range of this parameter variation was substantially larger, that it would be possible experimentally. Using the theoretical model described in Section 2, we carried out calculations of the mean (Sauter) diameter of bubble for each of these virtual liquids at five levels of the superficial gas velocity (Table 1). For each liquid we calculated the hold-up (ε) using Akita and Yoshida correlation [8]:

$$\frac{\varepsilon}{(1-\varepsilon)^4} = 0.2\left(\frac{gD^2\rho_L}{\sigma}\right)^{1/8}\left(\frac{gD^3\rho_L^2}{\mu_L^2}\right)^{1/12}\left(\frac{U_G}{\sqrt{gD}}\right) \quad (9)$$

The distribution parameters μ_{nl} and σ_{nl} we found for the minimum value of G_i (from Eq. (6)). Fig. 2 shows the log-normal distribution for virtual liquid no 1. as an example.

In the next step we calculated Sauter diameter using Eq. (3).

In this way we found the theoretical values of d_{32} for each of the 20 virtual experiments defined in Table 1.

4. Bubble diameter correlation

The Sauter diameter depends on the physical liquid properties and the gas velocity. This relation is supposed to have the form:

$$d_{32} = A\rho_L^{\alpha}\mu_L^{\beta}\sigma^{\gamma}U_G^{\delta} \quad (10)$$

To find the values of the coefficients A, α, β, γ and δ a linear regression method was used. The following values were obtained: $A = 0.289$, $\alpha = -0.552$, $\beta = -0.048$, $\gamma = 0.442$, $\delta = -0.124$. The values of α, β and γ show very good agreement with van Direndonck correlation [9] (Table 2). The value of exponent δ shows also very good agreement with Pohorecki et al. correlation [4] for the nitrogen–cyclohexane system ($U_G^{-0.12}$). Finally, the theoretical correlation for the Sauter diameter in bubble columns reads as follows:

$$d_{32} = 0.289\rho_L^{-0.552}\mu_L^{-0.048}\sigma^{0.442}U_G^{-0.124} \quad (11)$$

Table 1
The properties of virtual liquids and gas velocity

Experiment number	Density ($kg\,m^{-3}$)	Viscosity (Pa s)	Interfacial tension ($N\,m^{-1}$)	Gas velocity ($m\,s^{-1}$)
1	1000	0.0005	0.015	0.0144
2	1000	0.0005	0.025	0.0144
3	1000	0.0005	0.05	0.0144
4	1000	0.0005	0.1	0.0144
5	1000	0.0005	0.15	0.0144
6	1000	0.0001	0.05	0.0144
7	1000	0.0005	0.05	0.0144
8	1000	0.0015	0.05	0.0144
9	1000	0.003	0.05	0.0144
10	1000	0.005	0.05	0.0144
11	500	0.0005	0.05	0.0144
12	1000	0.0005	0.05	0.0144
13	1500	0.0005	0.05	0.0144
14	2000	0.0005	0.05	0.0144
15	2500	0.0005	0.05	0.0144
16	1000	0.0005	0.05	0.005
17	1000	0.0005	0.05	0.0144
18	1000	0.0005	0.05	0.02
19	1000	0.0005	0.05	0.03
20	1000	0.0005	0.05	0.05

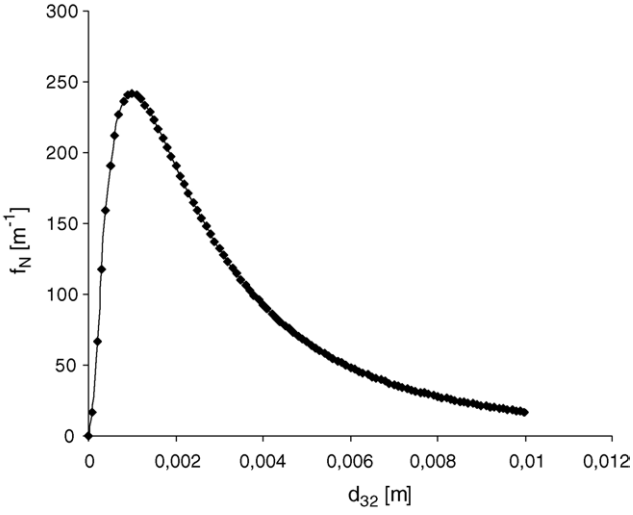

Fig. 2. Bubble size distribution function for experiment no 1.

5. Non-dimensional correlation

Relation (11) can be put into dimensionless form. As the existing dimensionless correlations give the bubble diameter in an implicit form (the linear dimension appears both on the left

Table 2
Influence of the liquid properties on the bubble diameter according to existing correlations

Correlation	Liquid density	Liquid viscosity	Surface tension
Hughmark [11]	$\rho_L^{-0.2}$	μ_L^0	$\sigma^{0.6}$
van Dierendonck [9]	$\rho_L^{-0.5}$	μ_L^0	$\sigma^{0.5}$
Akita and Yoshida [8]	$\rho_L^{-0.74}$	$\mu_L^{0.24}$	$\sigma^{0.5}$
Kumar et al. [12]	$\rho_L^{-0.25}$	μ_L^0	$\sigma^{0.25}$
Idogawa et al. [13]	ρ_L^0	μ_L^0	$\sigma^{0.08,a}$; $\sigma^{0.03,b}$
Idogawa et al. [14]	ρ_L^{-0}	μ_L^0	$\sigma^{0.20,a}$; $\sigma^{0.08,b}$
Wilkinson [15]	$\rho_L^{-0.45}$	$\mu_L^{0.22}$	$\sigma^{0.34}$

[a] $P = 0.1$ MPa.
[b] $P = 1.0$ MPa.

and on the right hand side of the correlation), we decided to use a new form of the correlation, derived directly using dimensional analysis.

The correlation has the form:

$$Fr = K\left(\frac{U_G^4\rho_L}{g\sigma}\right)^{1/2} \tag{12}$$

where the Froude number is defined as:

$$Fr = \frac{U_G^2}{d_{32}g} \tag{13}$$

To obtain the correlation in the form given by Eq. (12), one has to assume that in the first approximation:

$$\alpha = -0.5 \tag{14}$$

$$\gamma = 0.5 \tag{15}$$

$$\beta = \delta = 0 \tag{16}$$

Using linear regression method we determined the value of the constant K to be 0.6 (Fig. 3). Thus the final form of the correlation reads:

$$Fr = 0.6\left(\frac{U_G^4\rho_L}{g\sigma}\right)^{1/2} \tag{17}$$

The form of correlation (12) is identical as the van Direndonck correlation [9] for the cyclohexane, except for the value of constant K, which in the latter is $K = 1.05$.

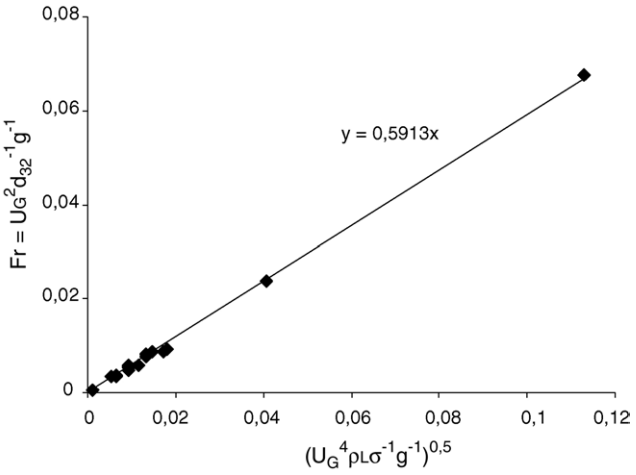

Fig. 3. The calculation of the constant in the non-dimensional correlation (17).

6. Comparison with experiment

The values of d_{32} calculated using both correlations have been compared with the experimental results of Bielski [2,10]. The experimental column used was a laboratory column 9 cm in diameter and 200 cm high (125 cm clear liquid head), operated at atmospheric pressure and low temperature, with seven different liquids: acetaldehyde, acetone, cyclohexane, isopropanol, methanol, *n*-heptane, and toluene. The bubble diameter was measured by a photographic method. The details of the experimental procedure have been described in our earlier paper [2]. The experimental Sauter diameters were compared with the theoretical Sauter diameters calculated using correlations (11) and (17). In calculation we used the actual liquid physical properties and gas velocities used in experiments.

The results of comparisons are shown in Table 3. As the non-dimensional correlation does not eventually contain the gas velocity, average values of bubble diameter obtained in experiments were used for comparison. The average error of d_{32} calculations from dimensional correlation is considerably smaller (2.80%) than that from non-dimensional correlation (16.40%).

In Fig. 4, a comparison of the bubble diameter d_{32} calculated from the dimensional correlation with the results of numerical or real experiments is presented. The agreement between numerical or real experiments and calculations from dimensional correlation is very good (average < 3%).

Table 3
Comparison of the experimental and theoretical values of the Sauter bubble diameter

Liquid	d_{32} (m) (experimental)	d_{32} (m) (dimensional correlation)	Error (%)	d_{32} (m) (average experimental)	d_{32} (m) (non-dimensional correlation)	Error (%)
Acetaldehyde	0.00426	0.00412	3.26	0.00368	0.00277	24.61
Acetone	0.00356	0.00346	2.92	0.00322	0.00291	9.54
Cyclohexane	0.00541	0.00526	2.77	0.00361	0.00303	16.21
n-Heptane	0.00316	0.00339	−7.27	0.00316	0.00288	8.94
Isopropanol	0.00344	0.00341	0.93	0.00335	0.00280	16.49
Methanol	0.00423	0.00430	−1.58	0.00280	0.00323	−15.25
Toluene	0.00376	0.00372	1.11	0.00398	0.00304	23.75

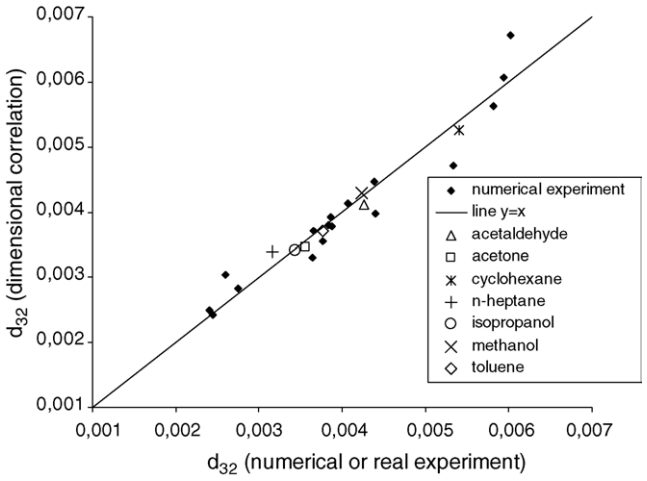

Fig. 4. Comparison of the bubble diameter d_{32} calculating from dimensional correlation with numerical or real experiment.

7. Conclusions

A simplified version of the theoretical model of Prince and Blanch [6] based on the population balance of the bubbles, was used to develop a theoretically sound correlation describing the Sauter mean bubble diameter in the main section of bubble column. A numerical experiment yielded values of exponents on the liquid properties (density, viscosity, surface tension) and the gas velocity, which seem to be more reliable that those appearing in experimental correlations.

The values of the Sauter bubble diameter calculated from the correlation developed are in good agreement with experimental values, obtained in the laboratory column for seven real organic liquids. The correlation has the form:

$$d_{32} = 0.289\rho_{\mathrm{L}}^{-0.552}\mu_{\mathrm{L}}^{-0.048}\sigma^{0.442}U_{\mathrm{G}}^{-0.124}$$

or, in an approximate dimensionless form,

$$Fr = 0.6\left(\frac{U_{\mathrm{G}}^4\rho_{\mathrm{L}}}{g\sigma}\right)^{1/2}.$$

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